November 2012

MADALGO seminar by Zhewei Wei, Aarhus University

Space Complexity of 2-Dimensional Approximate Range Counting

Abstract:

We study the problem of 2-dimensional orthogonal range counting with absolute additive error. Given a set P of n points drawn from an n*n grid and an error parameter eps, the goal is to build a data structure, such that for any orthogonal range R, the data structure can return the number of points in P \cap R with additive error eps*n. A well-known solution for this problem is the eps-approximation. Informally speaking, an eps-approximation of P is a subset A of P that allows us to estimate the number of points in P \cap R by counting the number of points in A \cap R. It is known that an \eps-approximation of size O(1/eps log^{2.5} 1/\eps) exists for any P with respect to orthogonal ranges, and the best lower bound is Omega(1/eps log 1/eps).

The eps-approximation is a rather restricted data structure, as we are not allowed to store any information other than the coordinates of a subset of points in P. In this talk, we explore what can be achieved without any restriction on the data structure. We first describe a data structure that uses O(1/eps log1/eps loglog1/eps log n) bits that answers queries with error eps*n. We then prove a lower bound that any data structure that answers queries with error O(log n) must use Omega(n log n) bits. This lower bound has two consequences: 1) answering queries with error O(log n) is as hard as answering the queries exactly; and 2) our upper bound cannot be improved in general by more than an O(log log 1/eps) factor.

Joint work with Ke Yi